Solution to "Arrow-bic Excercise"

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Introduction

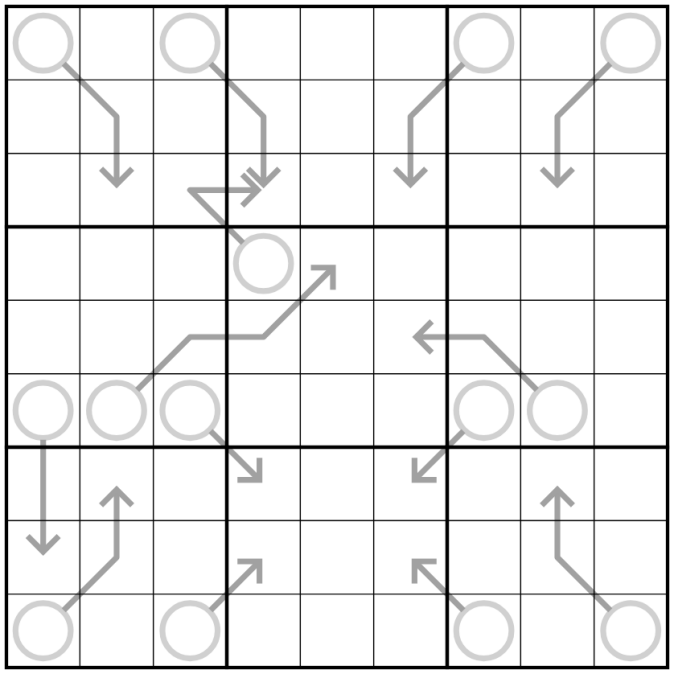
This is a solution for the sudoku puzzle "Arrow-bic Exercise", available to play at the following url: `https://sudokupad.app/az75jhr5ex`. Naturally there will be spoilers here, so try the sudoku before reading on! This guide will try to be fairly comprehensive, though we will not go through every little detail. Instead, some familiarity with sudoku puzzles is assumed, and we will only cover more involved steps. Note that we will only present one solution, there are very likely other approaches and variations!

Some quick notation: the rows, columns and boxes are numbered in the usual reading order, i.e. from left to right and then top to bottom. Cells will be labelled using 'RxCy' notation. So for instance, the cell in the third row and the fourth column is labelled R3C4.

If you just want to see the final solution, it can be found on the final page.

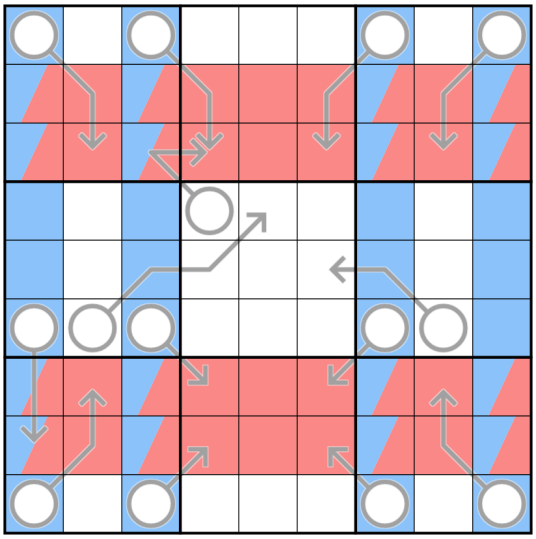
The Puzzle

The puzzle grid is below, and the rules are as follows. Normal sudoku rules apply (that is, place the digits 1-9 once in each row, column, and box). Digits along an arrow must sum to the digit in that arrow's circle.

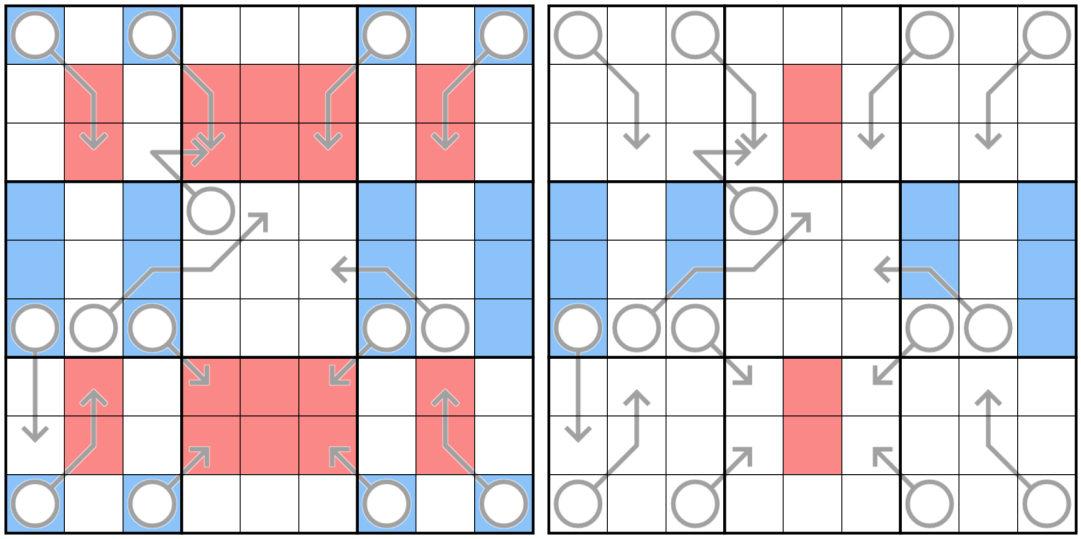


The Solution

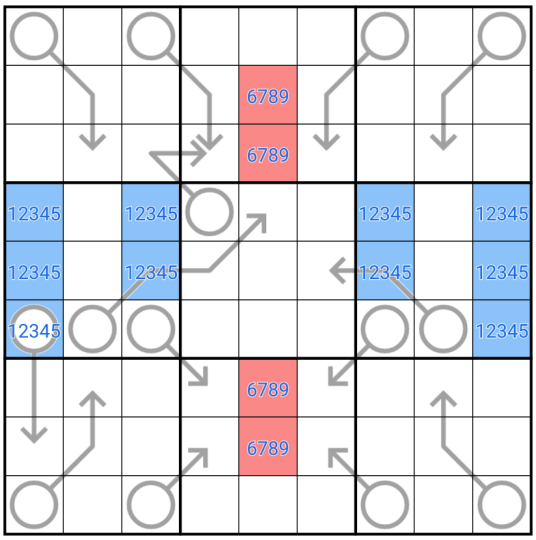
We start with a 'SET' argument: colour the columns 1,3,7,9 blue and the rows 2,3,7,8 red. Each colour has 4 sets of the digits 1-9, and so contain the same sum (4×45).



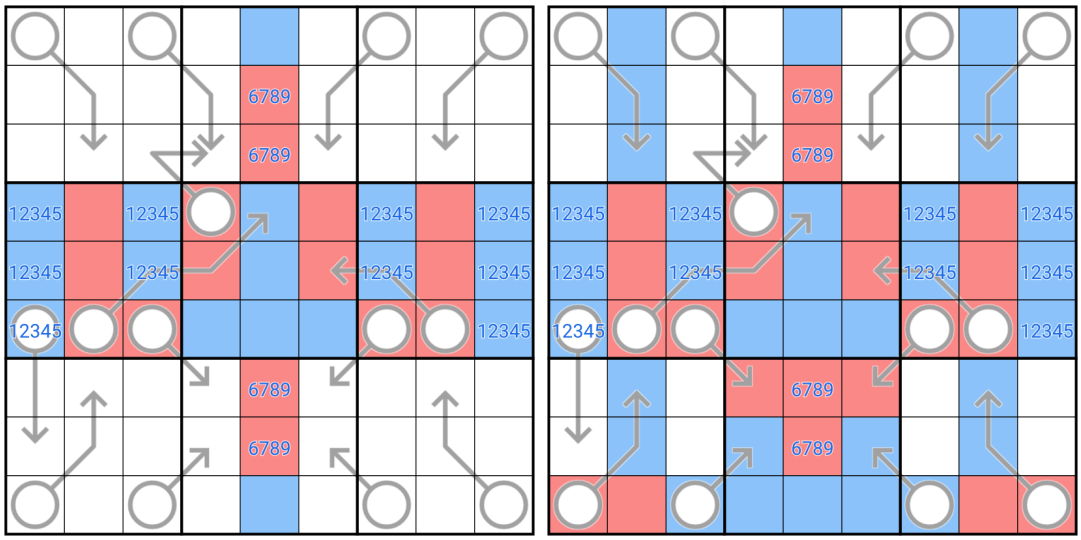
We can remove the squares in common, and any arrow overlaps, so that the sum of both colours is still identical.



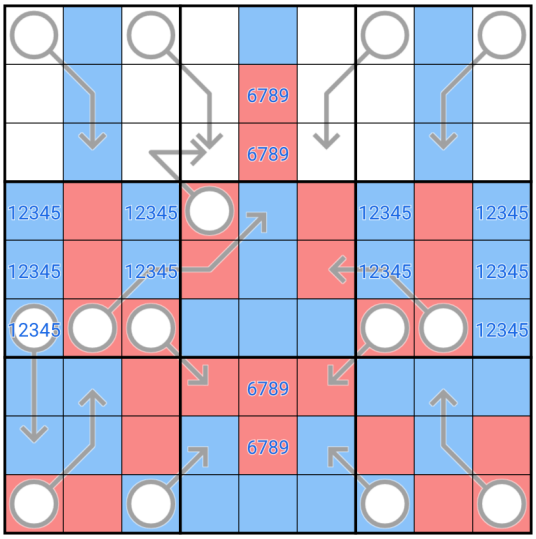
What remains of the red colour can sum to at most $6+7+8+9 = 30$. Conversely, the sum of the blue colour has to be at least $(1+2+3+4+5) * 2 = 30$. This immediately forces all red cells to be one of the 'high' digits (≥ 6), and forces the blue cells to be one of the 'low' digits (≤ 5).



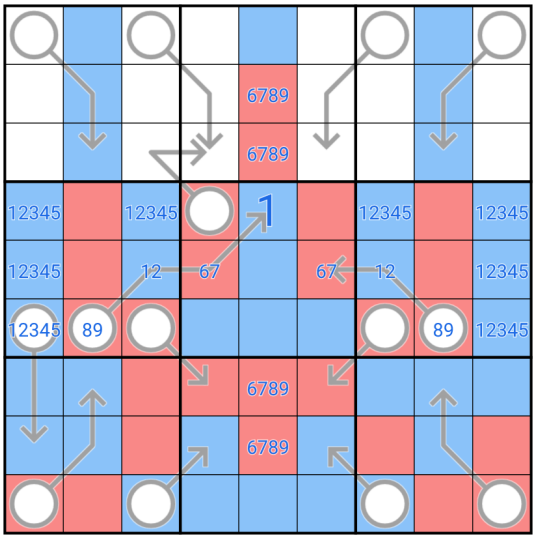
Going forward, we will colour any cells with a high digit red, and cells with a low digit blue. Note that any row/column/box has exactly 5 blue cells and 4 red cells. Using this, we can colour a few more cells.



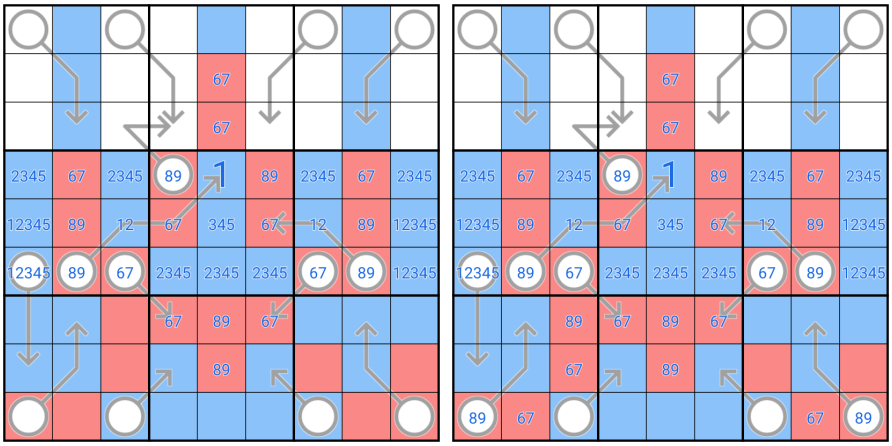
The circle in R6C1 must is low, and so the attached arrow cells must also be low.

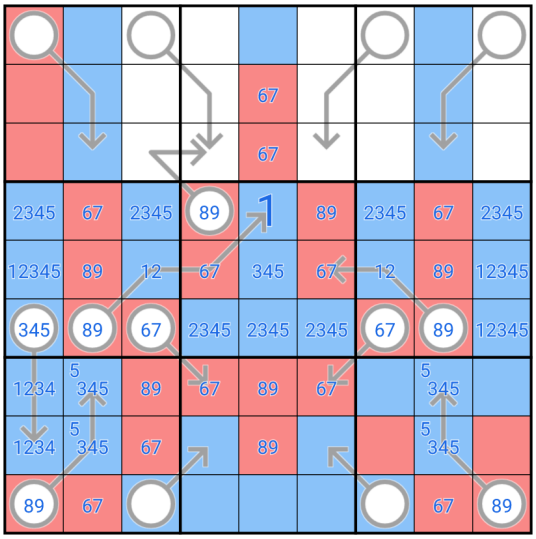


Next, notice the circles in R6C2 and R6C8. Their maximal value is $8+9=17$. On the other hand, we can work out the minimum value of their attached arrows. The cell at R4C5 could minimally be a 1. The pair of low (blue) cells in R5C3 and R5C7 is at least a 1,2 pair. And the pair of high (red) cells in R5C4 and R5C6 is at least a 6,7 pair. So the minimal value of the arrow cells is $1+(1+2)+(6+7)=17$ as well, and so we can fill in these arrows.

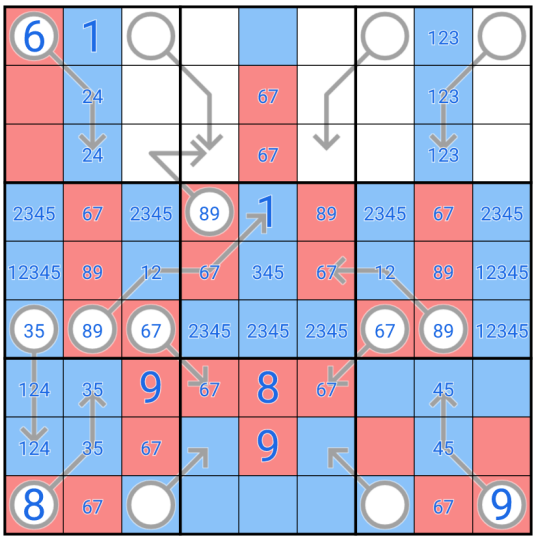


We continue with mostly straightforward sudoku logic.

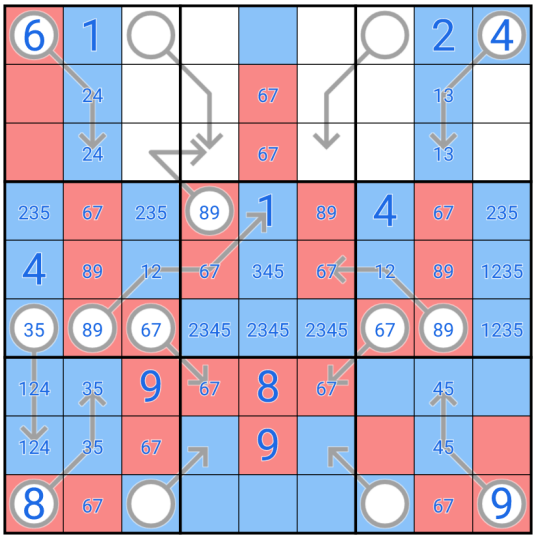




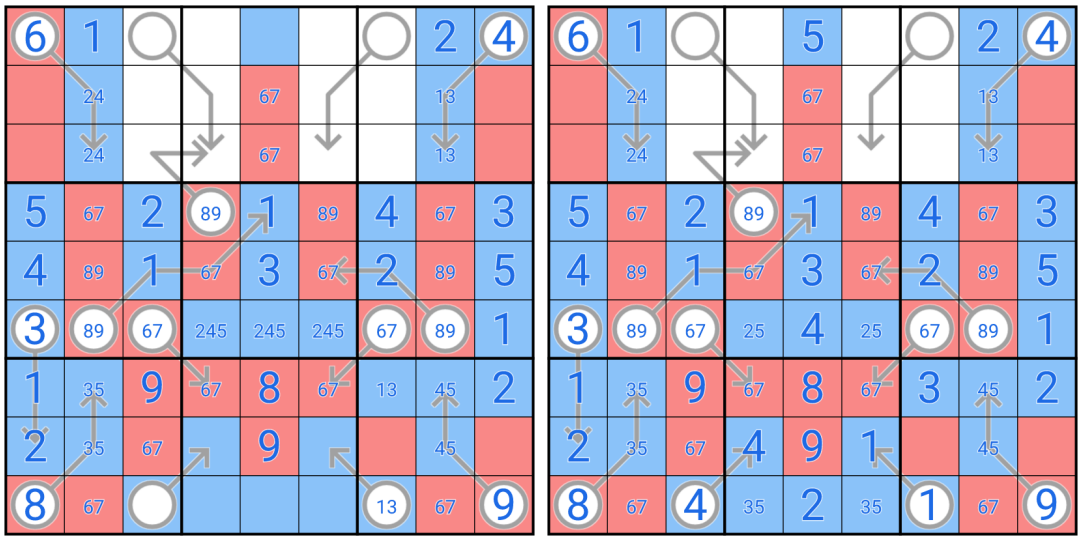
We again use this minimal/maximal trick on the arrows in the top and bottom left. Cells R1C1 and R9C1 are minimally a 6 and 8 respectively, which sum to 14. On the other hand, their corresponding arrows are all low digits, with a maximal sum of $2+3+4+5=14$.



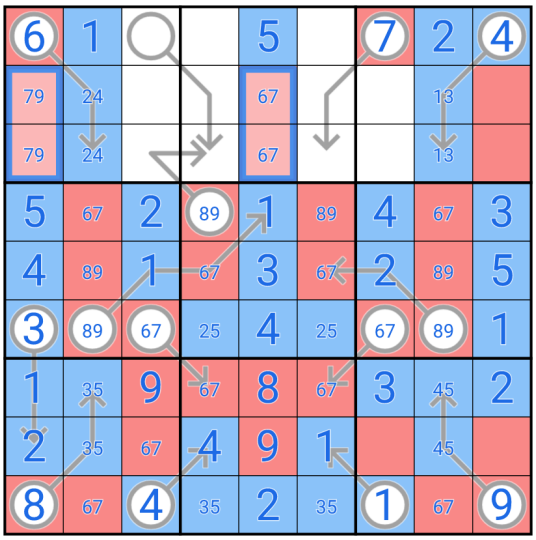
As shown above this forces a 9 in R9C9, and forces the digits in the eighth column of the third box to be a 1,2,3 triplet. Now consider the cell in circle in R1C9, which is the sum of two of these digits. It cannot be 3, as the triple in the eighth column contains a 3. It also cannot be 5, as this would force a 1 into R1C8, but we already have a 1 in R1C2. So it must be 4.



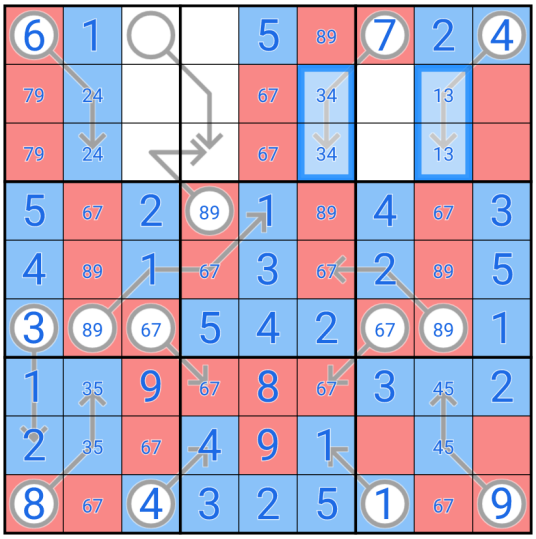
We continue with mostly straightforward sudoku logic.



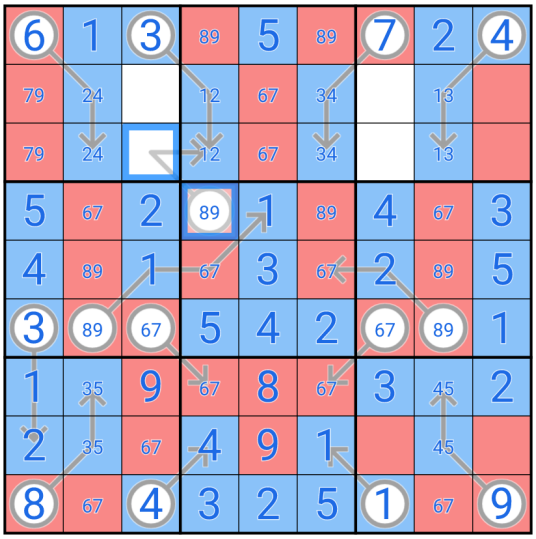
Next, notice that the 7 in column 1 must be in either row 2 or row 3. Likewise the 7 in column 5 must also be in one of these rows. This forces the 7 in box 3 to be in R1C7.



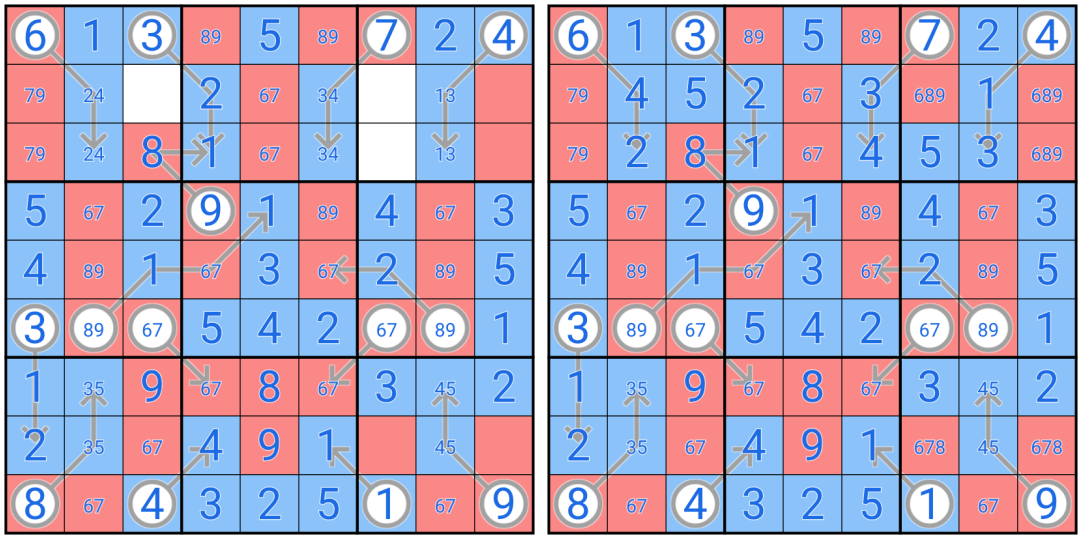
The associated arrow must therefore be a 3,4 pair.



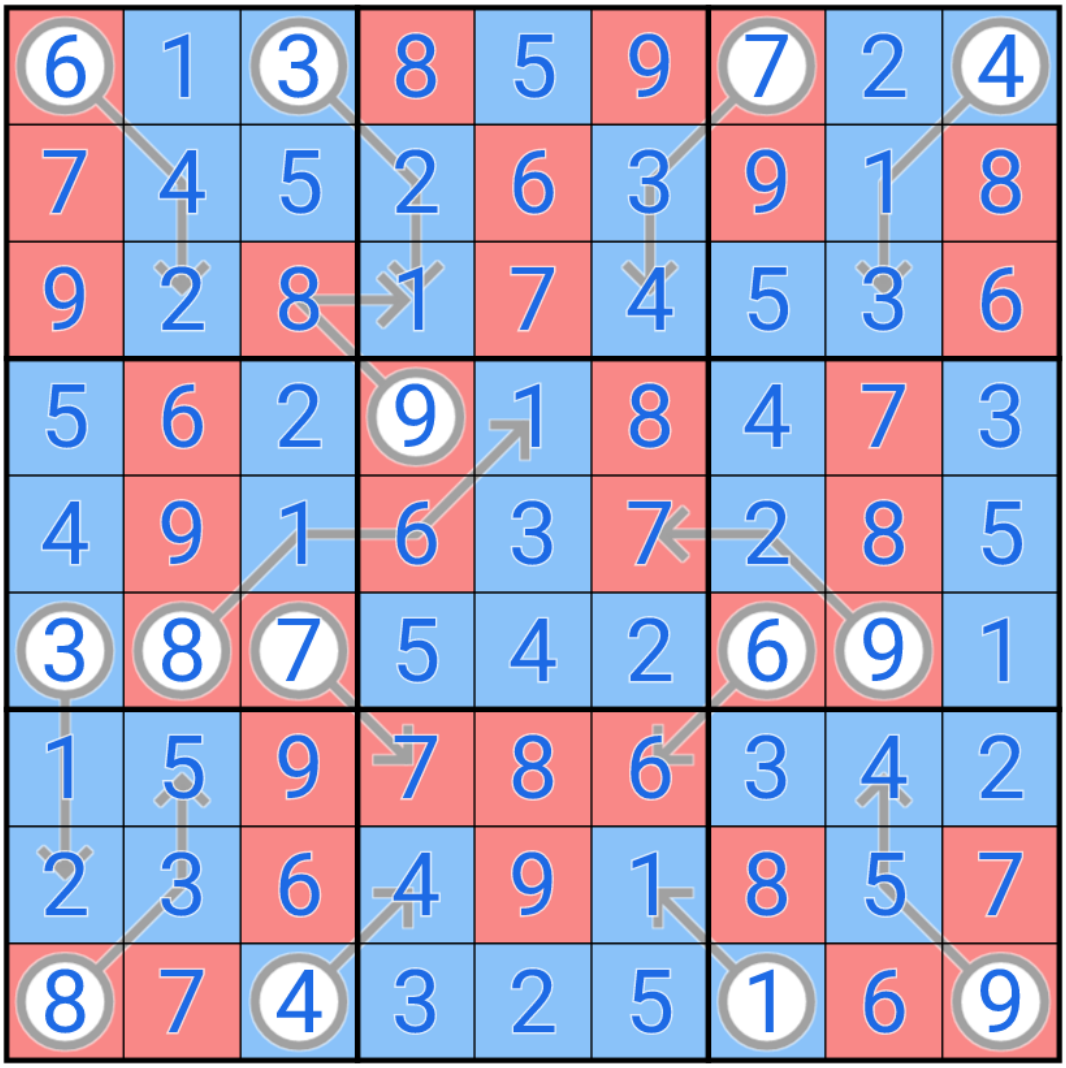
We can use same trick on the 3's in columns 6 and 8. They both must have a 3 in row 2 or row 3, so this forces the 3 in box 1 to be in R1C3.



Now R2C3 and R3C3 must be a 5,8 pair. The only way to make this compatible with the arrow attached to the circle in R4C4, is to place an 8 in R3C3.



This leads us to the final solution, which is on the next page. Thanks for solving, I hope it was enjoyable!


6 1 3 8 5 9 7 2 4
7 4 5 2 6 3 9 1 8
9 2 8 1 7 4 5 3 6
5 6 2 9 1 8 4 7 3
4 9 1 6 3 7 2 8 5
3 8 7 5 4 2 6 9 1
1 5 9 7 8 6 3 4 2
2 3 6 4 9 1 8 5 7
8 7 4 3 2 5 1 6 9